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ORIGINAL ARTICLE

Adaptive toolpath setup to reduce machining time when milling complex shape parts

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Abstract
This paper proposes an advanced algorithm to find the best setting of specific toolpath cutting parameters when using a milling tool with the circular cutting edge (e.g., ball-end, toroidal mills) to achieve the lowest possible milling time while obtaining the required scallop value for 3 + 2 axis milling operations. The proposed algorithm can effectively check the range of input parameters that has been set and determine the cutting angle and tool orientation to the machined surface in a combination resulting in the lowest machining time. As the algorithm is intended to serve as an additional user function for any CAM system, the second phase is used to effectively check the maximum height of the remaining material and adjust the stepover to achieve the desired maximum scallop value, while keeping the toolpath native to the specific CAM system. To verify that the required remaining material height has been obtained, a complex surface is machined as a section of the knee replacement part. Using the machined shape surface as an example, it was verified that the algorithm first found an input parameters setting that saves 11% in milling time compared to the most time-consuming parameter setting variant, and in the second (adaptive) phase, the algorithm further optimized the scallop calculation value to save another 19% in milling time, while the required maximum remaining material height on the workpiece surface was analyzed.

Keywords Toolpath · Cutting direction · Tool orientation · Scallop · Productivity

1 Introduction

The constant pressure to increase production efficiency to keep businesses competitive is resulting in cost reductions. In production processes, this is the daily work of the technologist, who must be able to efficiently adjust many parameters to ensure that the production operation is as cost-effective as possible and that it results in a quality product. A number of factors must be taken into account when setting the technological conditions of the machining operation: the material of the machined part and cutting tool, number of teeth, coating, type of cooling, feed per tooth, cutting speed, stepover, and depth of cut, as well as the tool's geometric parameters and, for complex shaped surfaces, the geometric constraints arising from the machined surface (e.g., the minimum internal radius of the surface). Many operations involved in milling complex-shaped parts are considered 3 + 2 axis milling, as they offer shorter machining times compared to continuous multi-axis milling, especially on heavy machine tools. Shi et al. presented a solution to automatize the setting of cutting conditions for 2,5D cutting operations (face milling, drilling, etc.) by using neural network [1]. Santhakumar and Iqbal proposed a neural network model to automatically predict the resulting parametres (e.g., time, roughness, temperature, and milling energy) and find the best setting of cutting conditions (e.g., feed rate, loop spacing, cutting speed) for 2,5D trochoidal toolpath [2]. When a toroidal tool is used to mill the complex shape part, it is also necessary to check for collisions between the tool and workpiece outside the contact points needed to calculate the toolpath itself. The type of machining operation (roughing, semi-finishing, or finishing) also influences how many of these parameters the technologist must set. Toolpath generation is based on computational algorithms that must be able to process a mathematically described complex surface in combination with a particular type of template that is used as a pattern (e.g., spiral, offset contour, straight back, and forth motion). The

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technologist determines the appropriate template type for the operation based on the main shape of the machined surface.

Naples and Yi [3] presented different types of templates for generating point milling operations for shape-complex surfaces, and it was found that combining free and extrapolated surfaces produces a smooth uniform surface whose overall continuity increases as the order of the extrapolated surface increases. There are also strategies to calculate the toolpath for point milling operation that can work with changing the cutting direction during the toolpath [4], calculate the toroidal tool orientation to the machined surface [5] and [6] calculate the toolpath using multiple milling tools [7]. More optimization methods used in the process of multi-axis milling of complex shaped parts can be found in [8], where, among other things, an algorithm for calculation of gouge free toolpath for toroidal tool is presented. Sharma et al. presented a new toolpath calculation method for toroidal tools to control and eliminate finding a tool position that would have two contact points between the tool and the workpiece [9]. This is done by using a method where two consecutive points in the toolpath are checked for a so-called spin-angle. Huo et al. proposed a method for calculating the ball-end mill toolpath based on finding the feed rate vector field at each point of the machined surface in the tangent plane to exploit the surface curvature to obtain the lowest possible tool pass density while achieving the desired remaining material height [10]. However, the surface is processed all at once. Abdulghafour et al. proposed an algorithm for ball-end mill machining that can recognize whether the machined surface is concave or convex and the toolpath is adequately divided, while a different toolpath stepover calculation is applied according to the given shape of the surface to achieve the desired scallop value [11]. This reduces machining time. Redonnet deals with optimal cutting direction when milling the complex shape surface using toroidal mill to achieve the lowest milling time [12], however the method is suitable only for three axis milling, without tilting the workpiece or the tool by two additional rotational axes. An analytical solution to calculate the optimal scallop value is presented by Redonnet et al. in [13]. Herraz et al. segmented the machined surface based on geometric similarity to achieve the toolpath with the lowest machining time by selecting a different toroidal tool motion direction on each surface segment [14]. However, the authors do not address the optimal tilting of the workpiece or the tool when milling on 5-axis machine tool by 3 + 2 milling strategies.

Finding the optimal setting for milling operations is a challenging task for the technologist, as there are many variables (cutting parameters), including tool selection (material, coating, number of teeth, helix angle, etc.). Therefore, authors focus on automating routine tasks (processes) that are unproductive for technologists and some optimal settings can be found, e.g., in terms of achieving the minimum time. Fountas and Vaxevanidis developed an algorithm that can automatically select the ball-end mill tool diameter based on achieving the minimum machining time and the minimum deviation of the remaining material on the machined surface found in a machining simulation [15]. The algorithm was designed by using three different toolpath calculation templates in the CAM system and incrementally fits different tool diameter values and iteratively performs the time and deviation calculation. Herraz et al. presented a method that selects whether a ball-end mill or toroidal mill is more suitable to achieve a lower machining time [16]. The method determines two main toolpath directions. One direction is more suitable for a ball-end mill and the other direction for a toroidal mill. The effective cutting diameters for both a ball-end mill and toroidal mill are also analyzed. Gdula discusses finding a mathematical relationship between the lead angle of the toroidal tool and the actual curvature of the part surface and the resulting surface roughness [17]. The model was developed for 5-axis machining of a turbine blade with a toroidal tool. It was found that the relationship between the lead angle and the actual curvature of the part surface has a large influence on the longitudinal roughness and the model can be used to find an effective adjustment of these parameters. Another principle is introduced by Manav et al., who used a multi-criteria optimization model to calculate toolpath for machining of complex shape surfaces by ball end mills [18]. This model calculates the toolpath points to achieve an optimal relationship between machining time, cutting forces, and scallop value.

Duvedi et al. proposed an algorithm that can control the toolpath of the toroidal tool so that the tool is always in contact with the final surface at a minimum of two or more points [19]. This takes advantage of the maximum cross section of the tool for machining. However, a disadvantage is the limitation in areas of the surface where the radius of the protrusion is smaller than the tool diameter. The method also does not take cutting forces into account and so does not guarantee that undercutting will not occur when the cutter is pushed away during multi-point contact. Ma et al. proposed an advanced method for finding the optimal motion vector of a ball-end mill along a complex shaped surface using an algorithm based on "sub-regional" processing [20]. A set of points is first created on a model of the machined surface, at which the optimal tool motion vector is again searched for using a local tangent plane to the machined surface, so that both the minimum deviation at a given point (chord error) and the minimum scallop value are achieved. However, the surface is then segmented to achieve smooth tool motions according to the calculated motion vectors, which are adjusted to produce continuous toolpaths. This method achieved a lower time than global surface processing.

Development of cutting tool shapes has also progressed, and adequate strategies for technological operations must be available. A tool shape has been invented that does not have a spherical tip; its tip is formed by the rotation of an elliptical profile. The cutting diameter as well as the actual cutting cross-section are thus determined by the contact point between the tool and the machined surface. Therefore, Li and Tang presented an optimization technique to calculate the toolpath that determines the best orientation of this specific tool at each point of the toolpath to maximize material removal, and in a second step, the subpath sections are segmented to obtain a uniform distribution of tool orientation change to achieve smooth motion [21]. Osan et al. analyzed the effect of milling strategy on the resulting surface roughness when milling a general surface with a toroidal mill tool [22]. Better roughness was achieved with climb milling than with conventional milling. Osan et al. discuss a comparison of surface roughness when using a ball-end mill and a toroidal tool in general surface milling [23]. It was found that the toroidal and ball-end mills achieved comparable accuracies (the toroidal slightly better), and the toroidal tool showed lower machining times (about five minutes in this case). Hendriko et al. analyzed the effect on the scallop value of choosing the helix angle of the toroidal tool cutting edge simultaneously with the change of the tool axis orientation when machining a complex shaped surface with a toroidal tool [24]. A method called Grazing Toroidal Approximation was proposed to calculate the scallop value. With this method, it was found that as the helix angle increases, the scallop value is reduced. At the same time, decreasing the tool orientation angle also decreases the scallop value. Segonds emphasizes the importance of feed rate to be considered when predicting the scallop when milling with toroidal tools [25], since the feed rate directly impacts the secondary scallop, which is inspected in the cutting direction.

Varga et al. investigated the effect of tool inclination on machined surface quality and effective cutting speed by evaluating surface roughness and topography [26]. Experiments demonstrated that different tool positions, with a constant feed rate, have an effect on surface quality, with the best result achieved at a tool tilt of 15°. Abdullah et al. focused on obtaining optimal cutting parameters to achieve minimum machining time when milling pocket type features, i.e., features that have a constant vertical axis profile [27]. A fitness function was developed in the MATLAB software based on finding the effective feed per tooth, cutting speed, and depth of cut settings for the roughing and finishing processes, and the Genetic Algorithm Toolbox was used to perform the computational experiment and the algorithm was validated in combination with Mastercam software. The comparison showed that use of the proposed algorithm resulted in a 3.48% machining time savings.

Kukreja and Pande presented a novel approach to develop a machine learning based system for selecting the best toolpath planning strategy for CNC machining (finishing) of complex shaped surfaces directly from a CAD model [28]. A new toolpath analysis module is introduced that evaluates toolpath quality with respect to three performance parameters: surface quality, toolpath length, and smoothness. This quality measurement technique is extensively tested for robustness and accuracy. The results show that the proposed data-driven model achieved 96.8% test accuracy.

A cutting parameter optimization solution based on an expert database linked to the shape features on a model combined with the determination of quality parameters using Product Manufacturing Information (PMI) was presented by Tonejca et al. [29]. Using machine learning, an efficient design of cutting conditions for a given part feature is performed, but the geometric aspect of toolpath design is not addressed. Mauther et al. presented a new concept for optimization of process conditions based on processing monitored data from a sensor-based toolholder [30]. This monitored data is then transferred back to the CAM system using an evaluation application and the cutting conditions are subsequently adjusted using a specific extension function in the CAM. This procedure is therefore applicable in repetitive production or when a test part can be used before real machining. However, the geometric aspect of the toolpath design is not addressed. Fountas et al. focused on the design of a genetic algorithm for optimizing technological parameters in toroidal tool milling [31]. The variable parameters are the lead angle (three values), tilt angle (three values), and tool diameter (three values). The proposed algorithm was used to find the most suitable setting to achieve the lowest machining time. The aim is to optimize the milling time and surface accuracy. Li et al. proposed a neural network model to find an optimal setup of cutting conditions (spindle speed, feed rate, cutting depth, and path pitch) to get the best possible results of machining time, energy consumption, and surface roughness while milling of complex shape surfaces using ball end mills [32]. Zhang et al. introduced a neural network model to predict surface errors based on continuous learning to improve computational time and accuracy [33]. It was verified that after the neural network has been trained by 15% of the complex shape samples, it predicts the surface errors with an absolute error of less than 1 μm.

The need to optimize toolpath parameters and automate the toolpath calculation also arises when repairing parts using hybrid technologies (a combination of welding and milling), when producing the parts using additive strategies [34] or when deburring by using robots [35]. Friebe et al. proposed a fully automatic system that can design the optimal toolpath based on the scanned shape of the part after welding the material at the repair site and the original model of the part [36]. From the scanned data, it was necessary to calculate both the vectors determining the appropriate tool orientation relative to the machined surface and toolpath shape. The method is suitable for point milling of complex shaped surfaces such as radial turbine blades. Thakur and Chauhan mentioned the importance of efficient generation of adaptive toolpaths for manufacture of implants [37]. This method is designed for milling toolpaths, and it is not conventional milling, but rather forming by milling. The authors developed an algorithm that can also design the path with respect to the deviation caused by material springing, thus reducing the root mean square deviation of the desired shape by approximately 11% compared to the original condition.

Dai et al. proposed an innovative method of generating adaptive toolpaths (similar to trochoidal paths), which guarantees a constant depth of cut in 3D space while designing the necessary tool lead and tilt angles to reduce the acceleration requirements in the rotational axes of the machine [38]. This milling method is suitable for five-axis milling of rotors with radial blades (blisks). The experiment showed a reduction of almost 30% in peak cutting force during machining. Another example of using an advanced algorithm to predict the machining accuracy of shape-complex parts is presented by Rudel et al. [39]. By implementing a model based on a digital twin of the process that can use the FEM module in a CAD/CAM system to predict the force interactions during machining of the part, the setting of cutting parameters can be optimized in toolpath sub-sections by dividing it into several sub-sections depending on the distance from the blade head to the blade root during machining. This resulted in optimal conditions to achieve increased precision in surface finishing. However, the toolpath was not modified.

The lead and tilt angles also have a large influence on the resulting cutting force levels [40] and [41]. Therefore, the influence of cutting forces needs to be included in the optimization when the technologist must address machined surface quality, which can be an issue when machining thin-walled parts or when machining with slim tools.

Summary related to the point milling operations of complex shape parts using toroidal milling tools:

I) besides the machining allowance, feed per tooth, cutting speed, cutting pattern to mill the surface (zig, zag, zig-zag, spiral, offset, etc.), toolpath density (either by the step-over or scallop parameter), lead angle and tool tilt settings, the technological conditions also include the setting of the main cutting direction. Finding the optimal setting for all these technological conditions is a very challenging task for the technologist by using the actual available SW tools;

II) when using a toroidal tool, the tool diameter and tool tip radius play a very important role, with the actual tool cutting diameter depending mainly on the tool lead and tilt angles and the shape of the machined surface. Thus, when machining a complex surface using 3 + 2 (axis) milling operations, it is not easy at all for the technologist to select effective tilting of the tool for a specific segment of the surface, as the tool is always in a different position relative to the surface normal of the workpiece during the machining operation and the contact area between the tool and the workpiece cannot be easily imagined throughout the whole operation;

III) there are several advanced methods proposed to calculate the toolpath with the optimal orientations of the tool to the the surface in each toolpath point to achieve the most productive milling operation. However, these methods deal with continuous five-axis milling, where the orientation of the tool relative to workpiece is changing continuously;

IV) some of the methods in III) are focused on the optimization of lead and tilt angles, which are kept constant to the surface normal when milling. The genetic algorithms are also developed including the tool diameter as another parametr to deal with during the optimization. However these methods can be applied only for continuous five-axis miling operations;

V) there are a few proposed methods to find the optimal cutting direction on the surface to achieve the lowest possible milling time while maintaining the required scallop. However, these methods are proposed for 3-axis machining while the tool axis is parallel to the Z axis of the workpiece coordinate system.

VI) actually, several methods deal with integrating genetic algorithms or neural networks model to predict surface errors based on continuous learning to improve toolpath computational time;

VII) it is not known, what a solution is integrated in the specific CAM system core to calculate the toolpath, so the user have no idea if the scallop limit value is used efficiently unless an analysis of the remaining material is performed. Since this is time-consuming to perform manually, it is not an efficient utilization of the technologist's capacity. Therefore this process has to be done automatically;

No definitive solution has been found that would allow the technologist to effectively determine an optimal combination of technological parameters—including machining direction and tool tilting relative to a specific surface section—in order to achieve a time-optimal setup for toroidal tools in 3 + 2 axis milling, regardless of the toolpath calculation core. The cutting conditions such as feed per tooth and cutting speed are related to the specific material of the workpiece and the specific milling tool chosen, so these parameters are not directly related to the shape of the workpiece. Therefore, the aim of this work is to develop a method that will automatically search for a combination of the tool orientation and the cutting direction over the machined surface to achieve the lowest machining time while achieving the desired machining roughness, regardless of the toolpath calculation core.

2 Proposal for automatic selection of cutting parameters

The goal is to propose a method to achieve process automation of obtaining a best combination of three technological parameters (toolpath direction and two angles defining the tool axis vector relative to the workpiece) to achieve minimum manufacturing time while achieving the desired roughness (scallop limit value) after 3+2 axis milling. All three of these parameters have a major impact on achieving the effective tool contact area of the toroidal tool and hence on the overall machining operation time. The toolpath direction parameter can be set only when parallel planes pattern is used to calculate the toolpath. To technically implement this solution, all possible variants of the combinations of these technological parameters must be addressed.

2.1 Variable parameters

One of the main technological parameters that influence the setting of the 3 + 2 axis machining operation is the cutting direction of the operation, or the feed rate direction. The cutting operation can be zig, zag, or zig-zag and the machining direction is then set by the angle from a vector defining the sense of the coordinate system axis. The example shown in Fig. 1 shows two cutting direction angles for a point machining operation (left: 180° from the X-axis, right: 135° from the X-axis).

Two other technological parameters that have a significant influence on the effective contact area between the tool and the workpiece are the lead angle and tilt angle. An example of setting a machining operation with the toroidal tool lead and tilt angles when machining a shaped surface is shown in Fig. 2. Figure 2 also shows that the lead angle determines the tool orientation in a plane parallel to the feed direction, and the tilt angle determines the tool orientation in a plane perpendicular to the feed direction. Since the lead and tilt angles are defined relative to the normal vector of the surface to be machined, they remain constant during continuous five-axis milling (in most cases). However, in 3 + 2 milling, these angles change continuously due to the fixed tool orientation in the workspace. Based on this, we propose using the notation lead_z angle and tilt_z angle in this manuscript to indicate that these angles are defined relative to the Z axis and remain fixed to it.

2.2 Setup of parameters

To automate the selection process for these three technological parameters to achieve the least time-consuming operation, the settings of these three technological parameters

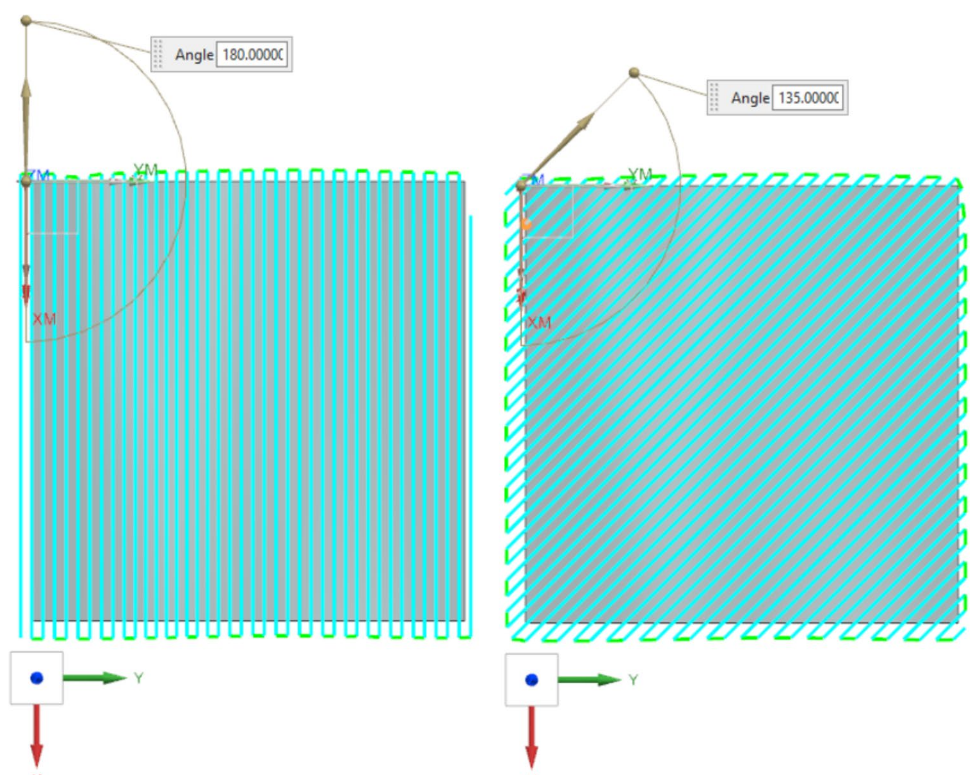

Fig. 1 Example of point milling cutting direction (left: 180° from the X-axis, right: 135° from the X-axis)

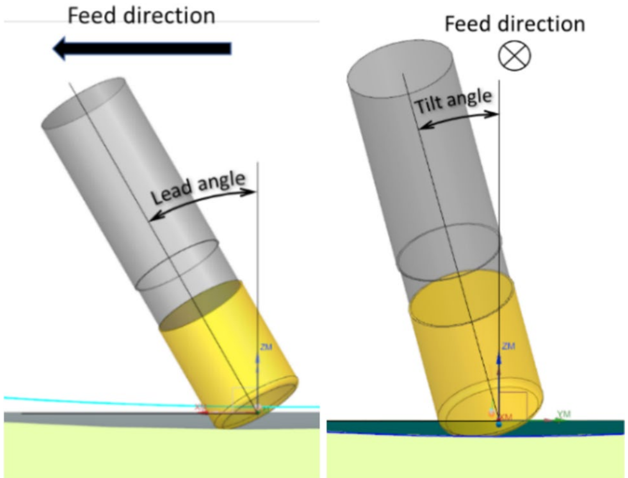

Fig. 2 Example of lead angle (left) and tilt angle (right) for a point milling operation

must be adapted so that they can be controlled directly by value and not manually by selection, as is common in CAM systems. An example of a shape-complex surface created by cutting off an ellipsoid from the face of a cylinder will be used to illustrate the setup; see Fig. 3. Figure 3 shows the main dimensions determining the design of the shape-complex surface.

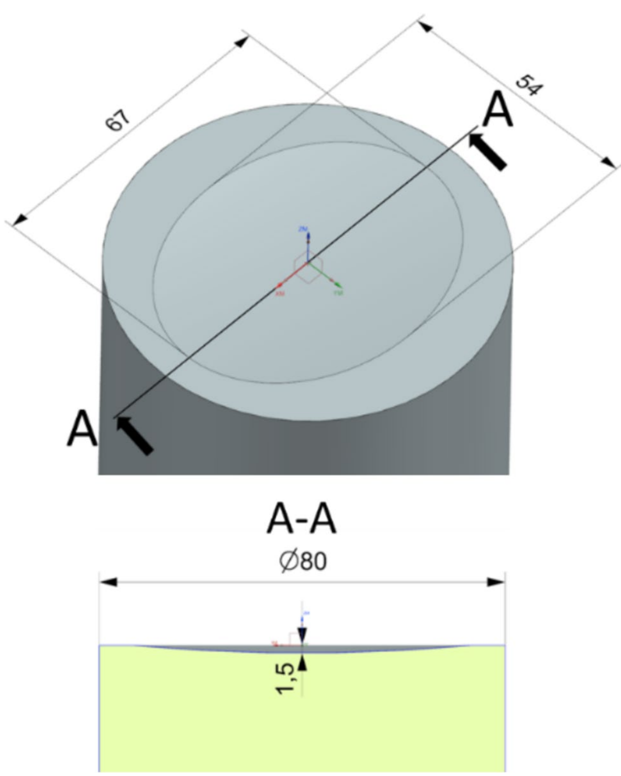

Fig. 3 Complex shaped part used for point milling setup demonstration

2.2.1 Cutting angle

The cutting direction in this case is defined by the angle from the X-axis. The definition of the direction of motion is made through an auxiliary sketch, independent of the part model, so that the lead_z angle and tilt_z angle can also be related directly to the tool cutting direction (feed direction). Using this sketch, the cutting direction angle (α) in the XY plane α is given, which can be characterized by a unit vector that only has components in the X and Y axes in this definition plane, i.e., $\vec{f} = \left[f_x, f_y, 0\right] = [cos\ \alpha, sin\ \alpha, 0]$. In Fig. 4, this vector is marked as feed direction. This figure also shows two variations of the angle setting of the tool sense of motion.

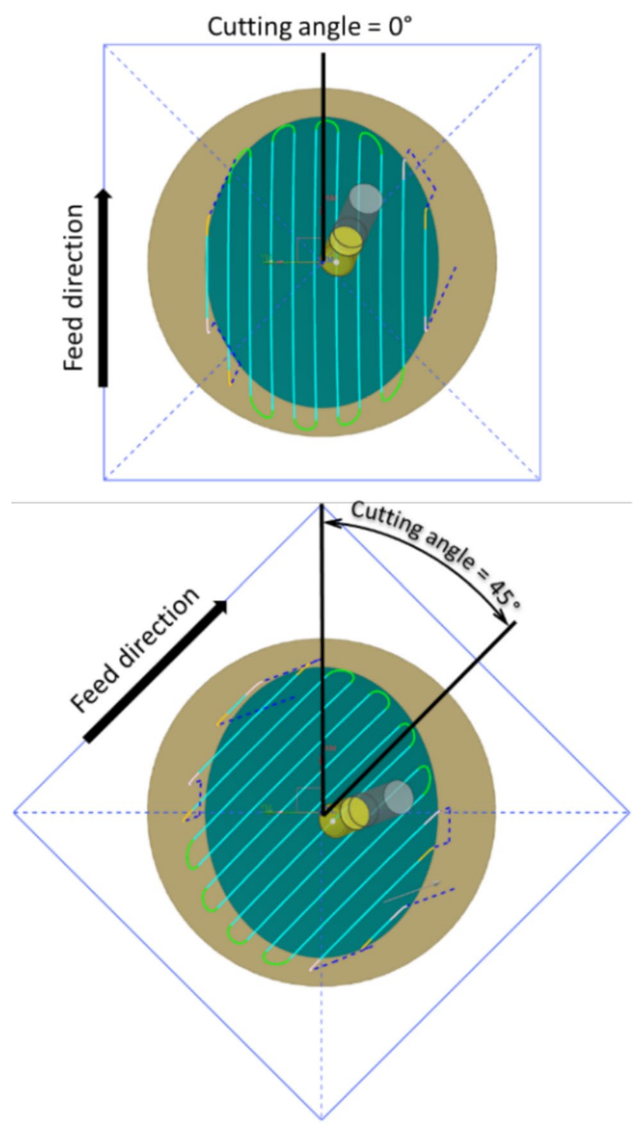

Fig. 4 Example of two cutting angles of point milling operation (top: cutting angle = 0°, down: cutting angle = 45°)

2.2.2 Definition of lead_z and tilt_z angles

The above vector $\vec{f}$ is also the axis along which plane τ is oriented, defining tilt_z angle φ (see Fig. 5). This plane is determined by Eq. (1).

$$\tau : a_1 \cdot X + b_1 \cdot Y + c_1 \cdot Z + d_1 = 0 \tag{1}$$

Vector $\vec{f}$ is orthogonal to vector $\vec{c} = [c_x, c_y, 0] = [\cos(\alpha + 90), sin(\alpha + 90), 0]$, while vector $\vec{c}$ is also the axis along which plane defining lead_z angle γ is oriented (see Fig. 5), and this is determined by Eq. (2).

$$\rho : a_2 \cdot X + b_2 \cdot Y + c_2 \cdot Z + d_2 = 0 \tag{2}$$

The vector that represents the tool tilt_z angle is marked as $\vec{t}$ in Fig. 5. Vector $\vec{t}$ results from tilting the Z-axis by an angle φ about vector $\vec{f}$ and therefore $\vec{t} = [sin\varphi \cdot \cos(\alpha + 90), sin\varphi \cdot sin(\alpha + 90), cos\varphi]$. The vector that represents the tool lead_z angle is marked as $\vec{l}$ in Fig. 5. Vector $\vec{l}$ results from tilting the Z-axis by an angle γ about vector $\vec{c}$ and therefore $\vec{l} = [sin \cdot \cos\alpha), sin\,\gamma \cdot sin\,\alpha, cos\,\gamma]$.

Then the coefficients of plane can be calculated as the vector product of $\vec{f}$ and $\vec{t}$ (3).

$$[a_1, b_1, c_1] = \vec{f} \times \vec{t} \tag{3}$$

Similarly, the coefficients of plane can be calculated as the vector product of $\vec{c}$ and $\vec{l}$ (4).

$$[a_2, b_2, c_2] = \vec{c} \times \vec{l} \tag{4}$$

The coefficients d_1 and d_2 are calculated by using a given toolpath point and the equality of the two equations of the planes at that point. The final step is to calculate vector $\vec{e}$, which represents the resulting tool orientation vector when lead_z and tilt_z angles are applied. Vector $\vec{e}$ is obtained as the intersection of planes and.

2.3 User setting of the range values of variables

Thanks to this mathematical coupling of the three parameters (cutting angle, tool lead_z angle, and tool tilt_z angle), any number of combinations of these parameters can be calculated to achieve minimum machining time. Another user-defined input parameter is the parameter determining the roughness of the machined surface. In CAM systems, this is either a parameter determining the spacing between tool passes (stepover) or a parameter determining the maximum height of the remaining material (scallop). The stepover parameter cannot be directly derived by the the technologist from the roughness parameters used in the definition on the production drawing (Ra, Rz), but the scallop parameter is already very close to the Rz value, which can be derived from the Ra parameter [Pešice 2023]. Since the toroidal tool does not have the same contact conditions at each point of the machined complex surface, CAM systems cannot accurately control the scallop parameter when generating

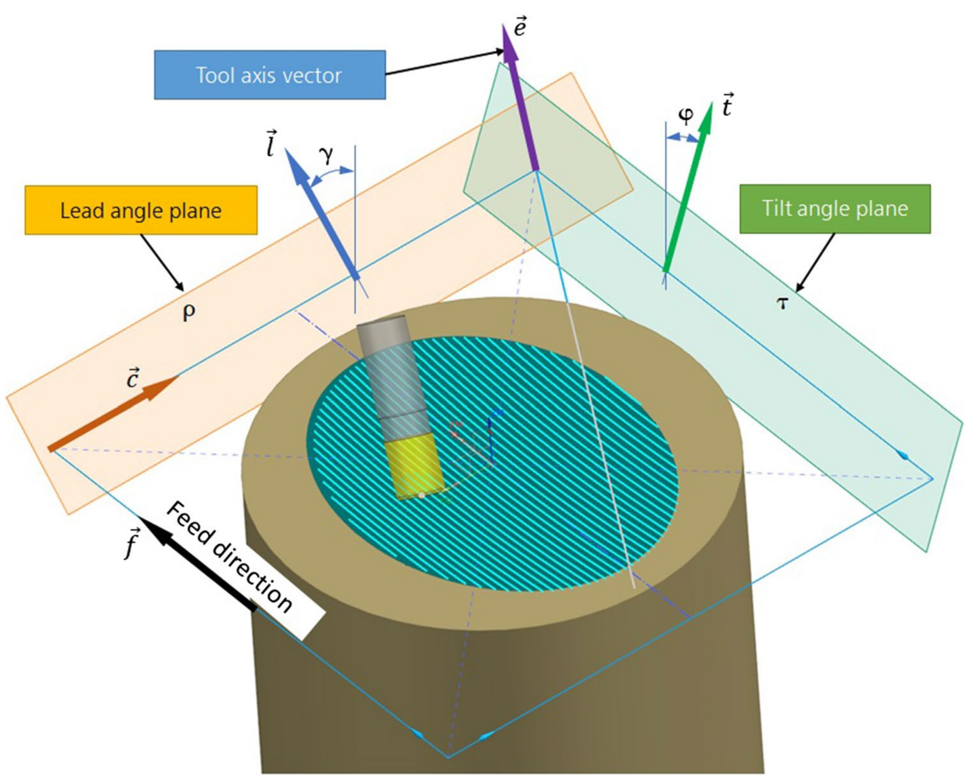

Fig. 5 Schematic representation of the planes and vectors used to obtain the tool axis vector

such a machining operation. Therefore, after generating the operation, it is always necessary to analyze the height of remaining material on the machined surface to check the maximum scallop value achieved in order to successfully set the conditions according to the roughness requirements to produce the part. Therefore, the automated process must take this check into account.

In terms of calculation efficiency, either the standard combinatorial approach can be used, or various advanced methods can be used to speed up the calculation [Abdullah 2023]. Of course, limiting the input values of the minimum and maximum range of the search parameters and the increment value of these parameters can also be used to limit the input values for the user.

When generating this operation, the CAM system must of course also check the minimum and maximum permissible angles (lead_z and tilt_z) to avoid undercutting the surface. To determine if the tool cannot remove the necessary amount of material in terms of its geometry and the specified angle parameters (material remains on the surface) in a certain part of the surface, the remaining amount of material is checked. Since these parameters are directly linked to the way the toolpath type is generated (according to the CAM system core), this check needs to be done during toolpath generation.

2.4 The principle of algorithm functionality

Since there is still no solution available to automatically select the most efficient toolpath parameter setting to achieve the minimum machining time, an algorithm was designed to eliminate the above-mentioned shortcomings, regardless of the toolpath calculation core. The main structure of the algorithm is shown in Fig. 6, where it can be seen that once the user inputs have been received, the i-th combination of input parameters is first calculated and then the common routine is started, i.e., the toolpath calculation using a preset operation in the CAM system, which also calculates the milling time (milling_time_i). The function to calculate the current i-th combination can be based on a common combination algorithm. After checking whether the toolpath calculation was successful (whether errors were not reported during the calculation, such as collision of the tool with the machined surface), the function of checking the remaining amount of material on the workpiece surface is started, which is a common function of the CAM system and also evaluates the maximum height of the remaining material (based on the difference of the 3D models) and this value can be saved (here as max_RMH_i). This value is compared with the desired scallop value, and if this value is exceeded, this parameter setting is "forgotten" and the next iteration is approached, i.e., the next i-th combination of input parameters. However, if the max_RMH_i value is lower than or equal to the

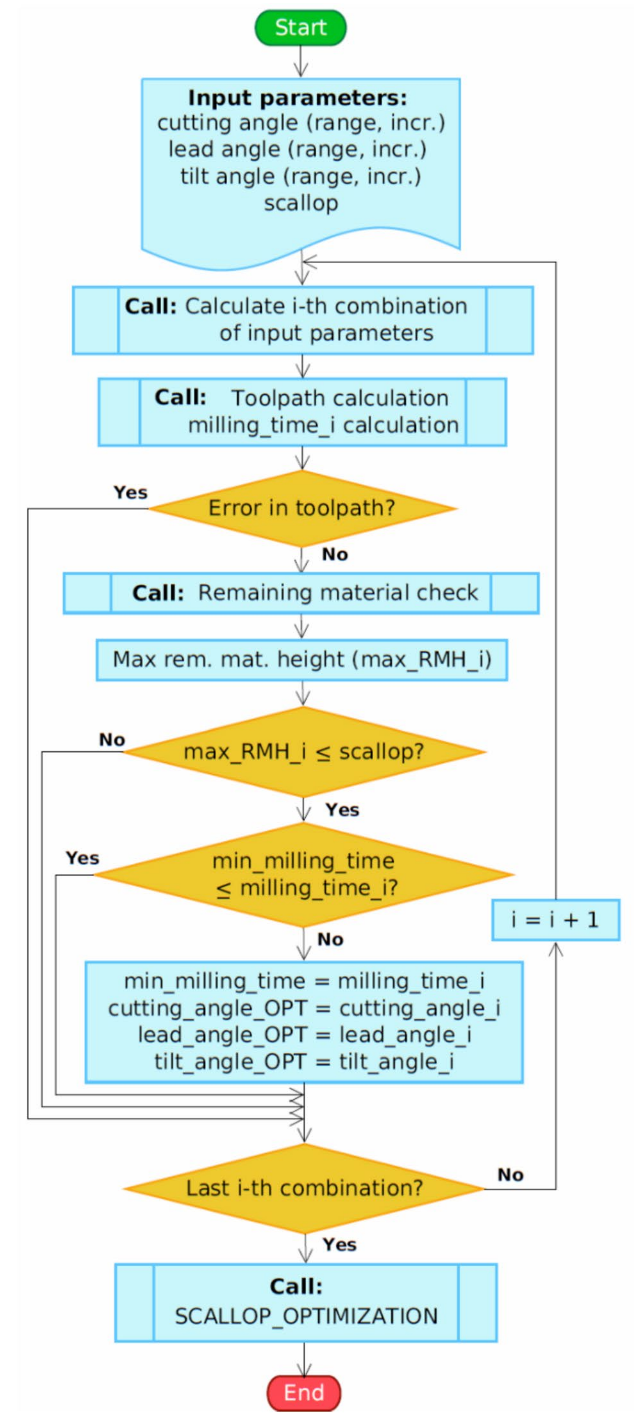

Fig. 6 Diagram of the automatic process to find the best toolpath parameters

scallop value, then it is further determined whether the milling time of the current path (milling_time_i) is lower than the previous lowest machining time, and if so, the current i-th value of the input values is saved as the best toolpath setting parameters. The algorithm can be used not only for toolpaths based on the parallel planes pattern, but also for other patterns (such as spiral, offset) when using only tool orientation (tilt_z and lead_z angles) and the max. scallop value as input parameters.

However, a very important section of the algorithm occurs after the best combination of input parameters has been found, i.e., after the last combination of input parameters has been checked. This second (adaptive) stage of the method can be seen in the flow diagram in Fig. 7. It has been observed that under certain cutting condition settings, the CAM system calculates a toolpath where the maximum remaining material height exceeds the specified scallop value. Conversely, for other specific settings, the toolpath results in a significantly lower remaining material height than the maximum scallop value (see Fig. 12). This adaptive process will ensure that the calculated scallop_OPT value is reset to a value that ensures the actual desired scallop value on the surface is met (in fact by adjusting the stepover, but

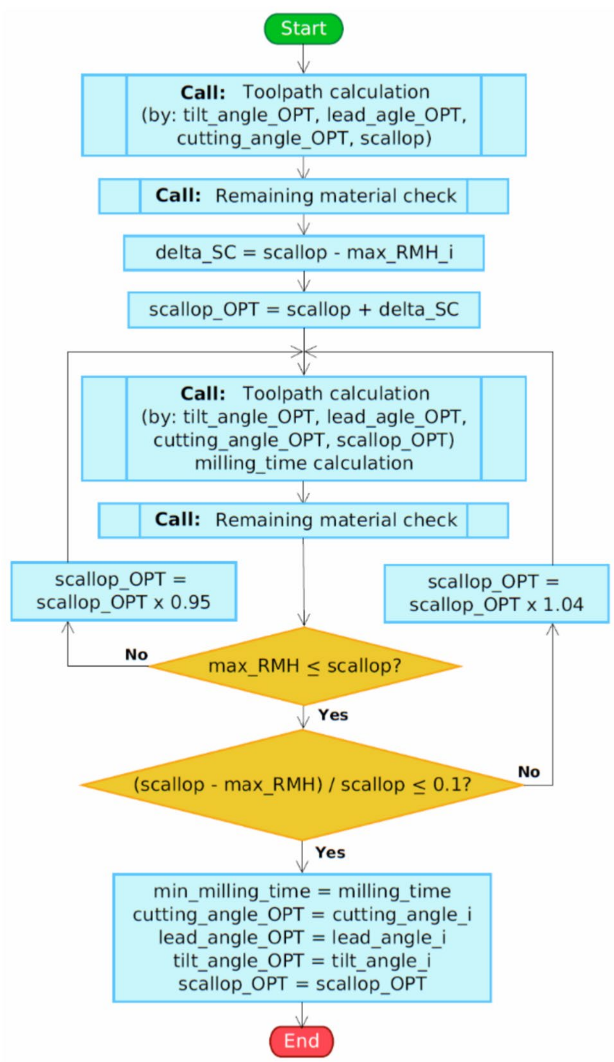

Fig. 7 Diagram of the "SCALLOP OPTIMIZATION" function

controlled by the scallop_OPT value) and thus the effective use of this limit value, further reducing the milling time. The algorithm starts by first running a standard function to generate a milling operation based on the best parameters that were determined, and then running a function to verify the maximum height of the remaining material, based on which the value (as opposed to the maximum desired scallop) is then calculated, by which the calculation value of scallop_OPT is increased in the next step. After modifying the value of scallop_OPT, the calculation of the toolpath is started again and the check of the maximum height of the remaining material (max_RMH value) is started. If the max_RMH value exceeds the permitted scallop value, the calculated scallop_OPT value is reduced by 5% and the toolpath is recalculated and max_RMH checked. If max_RMH is lower than or equal to the scallop value, then in the next step it is verified whether the difference in value is up to 10% of the scallop value, and if so, the process is terminated with this optimal calculated value of scallop_OPT. If the difference is greater than 10%, the calculated value of scallop_OPT is increased by 4% and the step of calculating the toolpath and checking the max_RMH value is performed again.

The graphical representation of the two algorithm phases are seen in Fig. 8. The CAM system strategy calculates the toolpath based on a scallop value defined by the technologist. As mentioned, the first stage involves verifying whether the toolpath calculation ensures that the maximum height of the remaining material does not exceed the predefined scallop value. Then the first stage selects the cutting conditions that result in the shortest milling time. Since the toolpath calculation in the CAM system can result in a significantly lower remaining material height than the specified scallop value, a second stage is applied to adaptively determine an optimal scallop value (scallop_OPT) using a remaining material check. The toolpath is recalculated to ensure that the final maximum remaining material height (max_RMH_fin) falls within the range: 0,9 scallop < max_RMH_fin ≤ scallop. It can also be observed that the final stepover value (s_{fin}) is greater than the initial stepover value (s_i). This approach maximizes the calculated scallop value (scallop_OPT) while ensuring it remains within the predefined scallop limit, ultimately reducing the total milling time.

2.5 Implementation into the CAM system

After designing the algorithm (Figs. 6 and 7) of the whole function, it was implemented in the Siemens NX CAD/CAM system in the form of an external dynamic-link library (mentioned as a DLL in Fig. 9) so that it could be run as an additional user function. In this case of the CAM system, the function is programmed in C# language and implemented using the NXOpen environment. Figure 9 presents a simplified scheme of the implementation of the function. From the

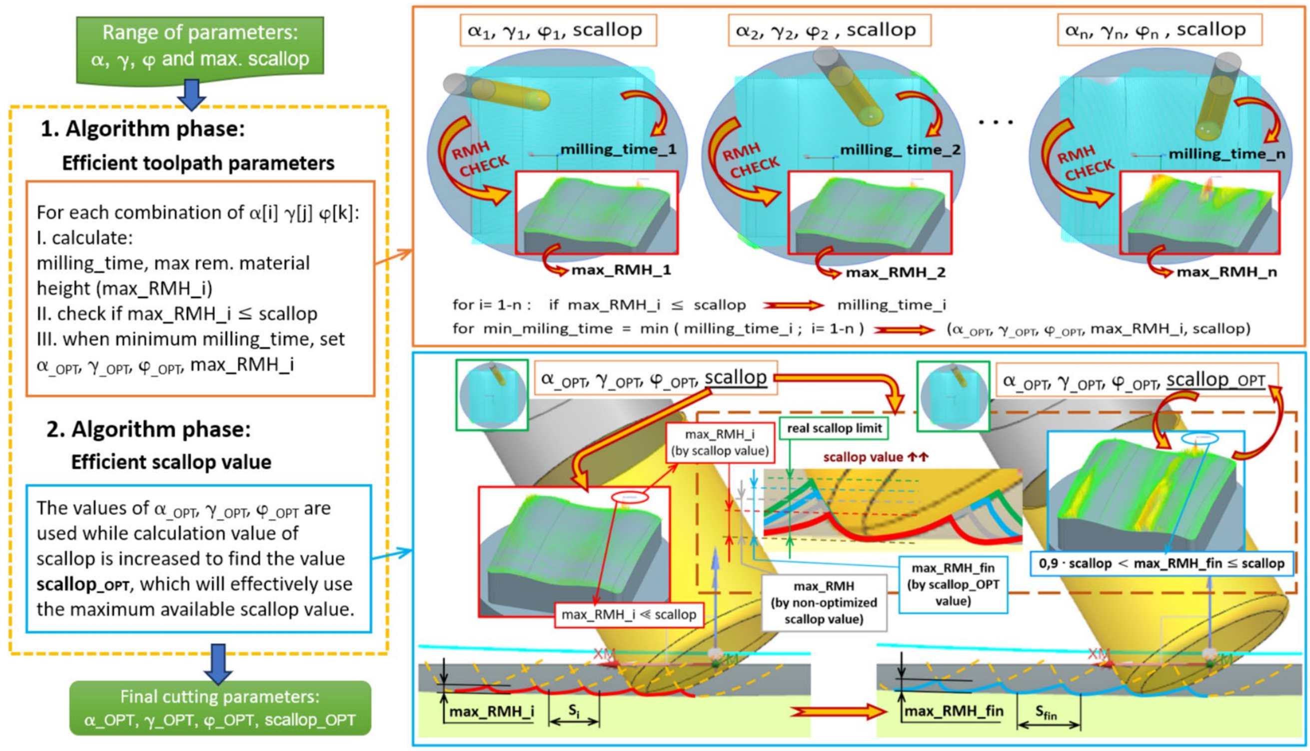

Fig. 8 Graphical representation of the two algorithm phases

point of view of user-friendliness, the function was designed not to change the technologist's standard tasks. Compared to the standard state, the technologist only determines the range of values (min–max) of the parameters: cutting angle, lead_z angle, tilt_z angle, and their increments. Other necessary parameter values such as scallop) are specified in the usual way during setting of the milling operation. After using this function, the user generates the NC program in the usual way by using the postprocessor.

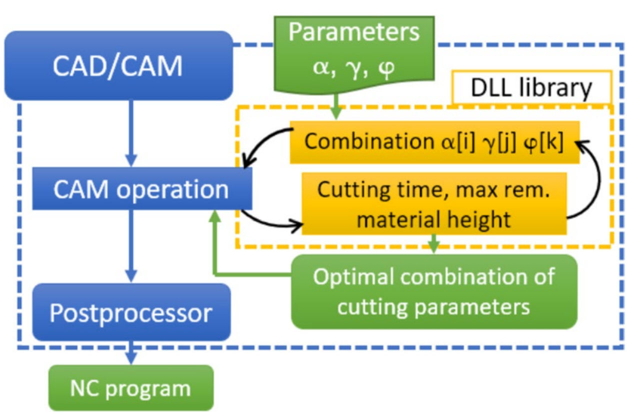

Fig. 9 Implementation into the CAM system

3 Results and discussion

Many automated processes are currently being developed due to increased automation in the technological preparation of the production of parts for the healthcare industry. In particular, parts for joint replacements, prostheses, or production molds for these parts are often individually adapted, as they are made from difficult-to-machine materials, and therefore their production time must be reduced. In this case, a real surface with a complex shape from a knee replacement part was used for verification (Fig. 10, left).

3.1 Testing surface

Multi-axis machining would be needed to machine the entire joint at once (Fig. 10, left), which would be quite uneconomical because the continuous interpolation of the rotational axes increases the machining time.

Therefore, a machining method using some surface sections is often chosen, where the rotary axes are only positioned to a constant value before machining and their position does not change during machining. Consequently, a wider section of the surface (Fig. 10, right) was selected to verify the functionality, which can be machined in several cutting directions. However, due to the suitably situated transition from the concave to convex section, a logical estimate can be made of the approximately optimal angle settings in

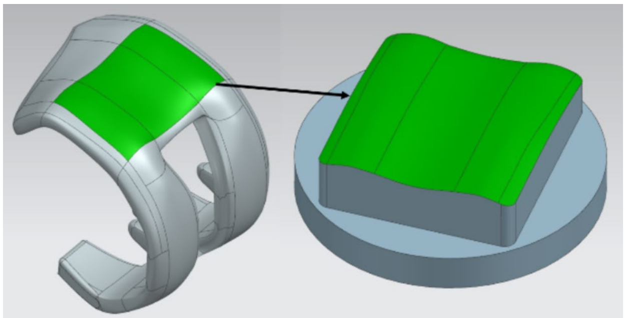

Fig. 10 Knee replacement as the testing part (left – whole part, right – specific surface section)

terms of time consumption. This can verify that the procedure is designed correctly.

3.2 Setup of cutting conditions

Since it is a complex shape surface and a toroidal tool is used for machining, the toolpath will be significantly different from the appearance of the machined surface.

The minimum tool diameter radius was analyzed on the surface model. This resulted in the selection of an 8 mm diameter tool with a 0.5 mm radius on the tool tip. Since it is a knee replacement made of metal materials, most often titanium or chrome-cobalt alloy, the main technological conditions for setting up the milling operation are adapted to this aspect. Therefore, a ETR080A4R05CF-6T05 tool (carbide grade IC903 with AlTiN PVD coating) with six teeth manufactured by Iscar was selected, and a cutting speed of 50 m/min and feed per tooth of 0.026 mm were selected based on the manufacturer's recommendation.

As it is a point milling operation with no smooth changes in the angular rotation axes' coordinates (they are fixly defined for the entire operation), the angles (lead_z and til_z t) are defined from the Z-axis. At the same time, it is also advisable to choose a particular leading and tilting angle range limit, due both to the shape complexity of the given surface and to limiting the range of values to obtain a limited number of data for better presentation of the outcomes. The basic limitation of toolpath input parameters can be seen in Table 1.

Based on the selected range of input values, toolpaths are automatically calculated, with a total of 18 different input parameter combinations, i.e., a total of 18 toolpath variants. The toolpath calculation tolerance is set to 0,001 mm and the maximum height of the remaining material (max. scallop) is set to 0.1 mm (at this point, given the lower number of toolpaths, a higher value is selected, which does not affect the verification). Each combination of input parameters (and thus the resulting toolpath) is marked as an "Index" with a sequence number for greater clarity and subsequent orientation in the text, figure, or table. An example of three toolpath variants for three different combinations of input parameters can be seen in Fig. 11. Figure 11a shows an example of a toolpath when the cutting angle (α) is set to 0°, Fig. 11b shows an example of a toolpath where the cutting angle is set to 45° and Fig. 11c shows a toolpath where the cutting angle is set to 90°.

Table 1 Setup of parameters for toolpath calculation

Parameter	Range	Increment	Count
Cutting angle	(0–90)°	45°	3
Lead_z angle	(10–30)°	10°	3
Tilt_z angle	(5–10)°	5°	2

3.3 Height of remaining material analysis

During this procedure, all 18 toolpath variants are automatically calculated and at the same time the maximum height of the remaining material on the machined surface is evaluated. First, a precise 3D model (STL format) of the workpiece is generated on the basis of actually calculated toolpath. This 3D model of workpiece is then compared to the 3D model of the part by a simple routine to analyze the maximum height of remaining material. On the basis of the analysis, information regarding the maximum height of the remaining material on the workpiece surface is added to the time data on the duration of the milling operation, which makes it possible to assess whether the toolpath was calculated correctly or incorrectly. Where there is an incorrectly calculated toolpath (the maximum height of the remaining material on the surface exceeds the permitted value), this toolpath variant is no longer included in the set of suitable toolpath variants from which the time-best toolpath setting variant will be selected.

Figure 12 shows six examples of toolpath variants where the values of the remaining material on the workpiece surface are plotted at the toolpath points, and at the same time the highest detected height of the remaining material on the workpiece surface is highlighted. The values of the height of the remaining material along the toolpath are shown here using a color scale, enabling a quick visual assessment of whether they are satisfactory or not. However, this is done for presentation purposes only; there is no need to do this step of coloring the toolpath points during the procedure. It is evident that the CAM system could not comply with the specified maximum height of the remaining material (scallop = 0.1 mm) and exceeded this value in some places of the given toolpath variant by almost 0.02 mm (e.g., Fig. 12b, e, f). It is also apparent that the color scale has been set so that all values above 0.1 mm are a distinct red color.

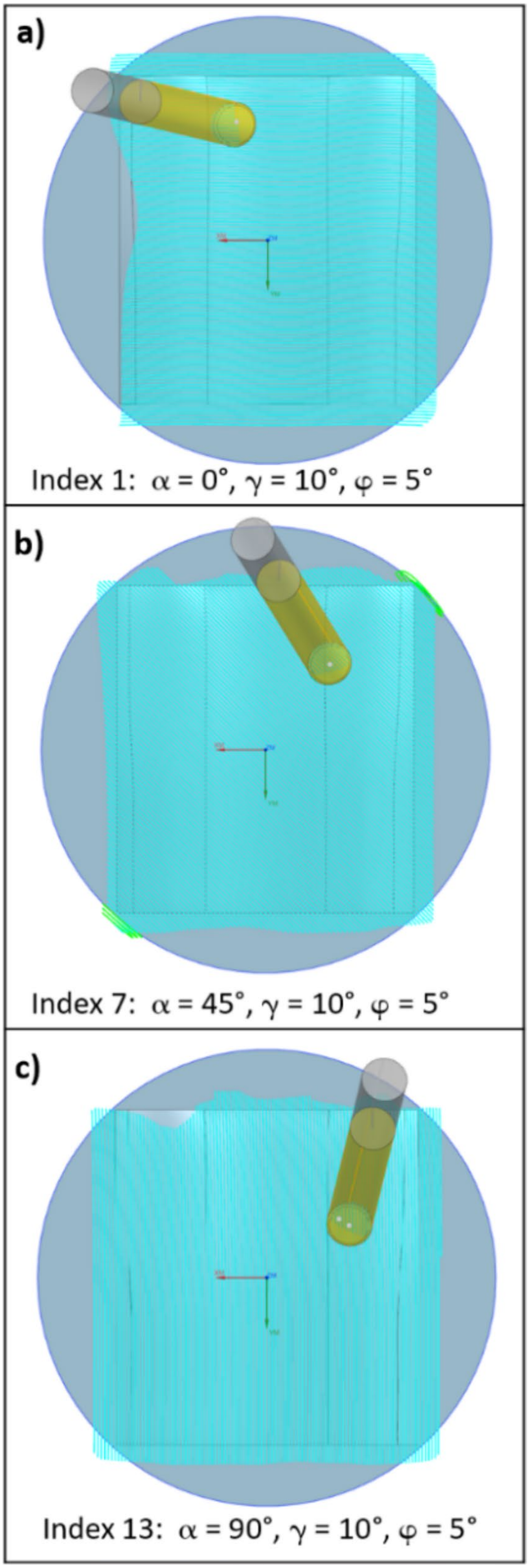

Fig. 11 Example of three different toolpath setups

3.4 Machining time comparison between CAM and CNC

A Kovosvit MAS MCU 700 VT-5X five-axis mill-turn machine tool was used for testing. The machine tool is equipped with a Heidenhain TNC640 control system and Step-Tec spindle with 24,000 RPM. The machine tool with the control system can be seen in Fig. 13. The drives of linear machine axes (X, Y, Z) are based on ball screws. The rotary axes A and C on the machine table are driven by rotary direct drives.

This machine tool is used to determine the real machining times of all the toolpath variants. This means that after generating the toolpaths, 18 NC programs were generated for this machine tool via the postprocessor, each containing a given toolpath variant. During testing, the feed rate was set to 320 mm/min and the spindle speed to 2000 RPM, which corresponds to the cutting conditions presented in subchapter 3.2. Thus, for each toolpath variant, the machining time is known from the CAM system, and after starting the NC program on the machine tool, the machining time corresponding to the given toolpath variant was obtained from the control system. A comparison of machining times is shown in Table 2 along with an indication of the relevant toolpath variant (Index), parameter setting and determined machining times from the CAM system (time CAM) and machining times from the control system (time CNC). This detection of machining times is primarily used to verify that the ratio between the times from the CAM system and CNC system is constant, i.e., that the minimum time can now be evaluated directly in the CAM system. The ratio between the detected times from the CNC system and CAM system was therefore calculated, i.e., the ratio "time CNC/time CAM". It is evident that the times obtained from the CNC system are longer than those from the CAM system due to toolpath processing and also starting up the spindle to the required RPM. From Table 2, it follows that this ratio is practically constant, therefore the machining time can be evaluated directly in the CAM system.

A comparison of the machining times of the calculated toolpaths shows that the shortest machining time was achieved by the toolpath variant marked by Index 17, while the longest machining time was achieved by the toolpath variant marked by Index 8; see Table 3. From these times it can be deduced that the time difference is 2.84 min, which is 11% less than the longest time. This time difference actually expresses the space that is available to find the best setting of the toolpath parameters to achieve the minimum machining time. For a larger area, or if a lower scallop value is required on this area (to achieve a smoother surface or lower roughness), very significant time savings will be achieved.

The usefulness of the first phase of the algorithm, which selects the best cutting angle, tilt_z angle, and lead_z angle to achieve the minimum machining time, was thus shown. Next,

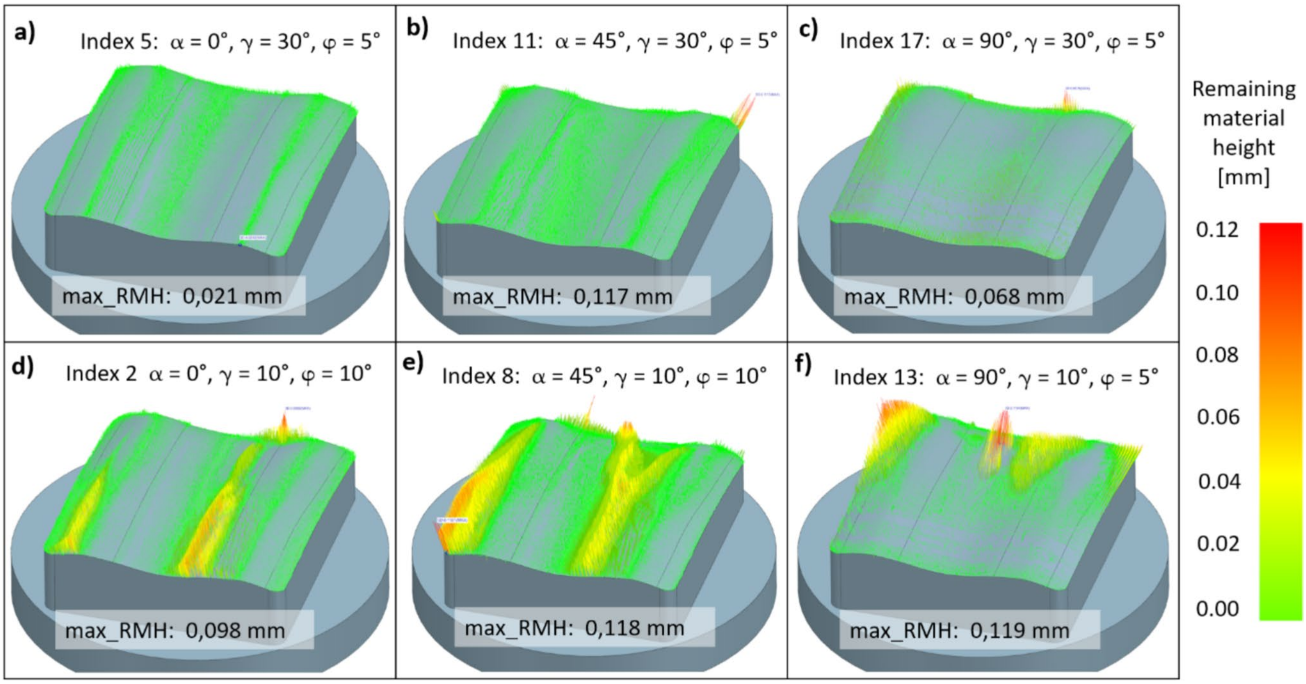

Fig. 12 Remaining material height analysis for six toolpath setups

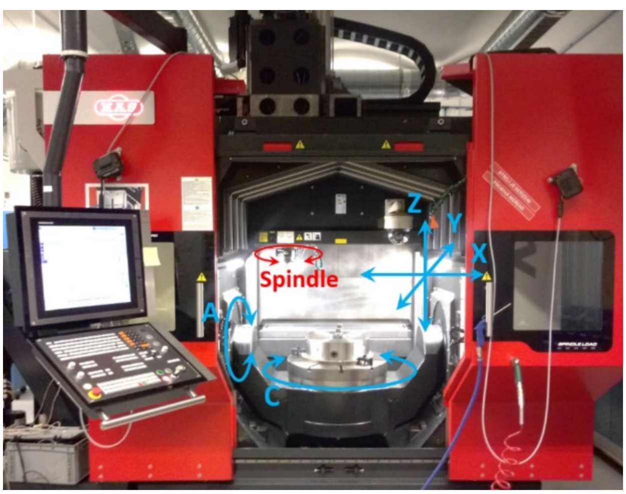

Fig. 13 Kovosvit MAS MCU700VT-5X machine tool with Heidenhain TNC 640 control system

the second phase of the algorithm begins, which is used to optimize the scallop calculation parameter so that the maximum permissible height of the remaining material is used to reduce the density of the toolpaths and thus reduce the machining time. As can be seen from Fig. 12c, the height of the remaining material on the machined surface is 0.068 mm, which is less than the required value (0.1 mm). Thanks to the "SCALLOP OPTIMIZATION" function (see Fig. 7) used in the second phase of the algorithm, the scallop_OPT calculation value will be optimized to 0.167 mm, which will cause the maximum height of the remaining material on the workpiece surface to be max_RMH = 0.093 mm (see Fig. 14), which is within a tolerance of 10% from the permitted scallop value. This resulted in a further machining time saving to 19.4 min, which is a 19% time reduction compared to the one carried out in the first phase.

3.5 Machining test, measurement, and evaluation

It is necessary to verify if the predicted scallop value is really achievable after real milling. The verification process to compare the max. predicted scallop value with the max. analyzed scallop value can be seen in Fig. 15.

It is not necessary to use the original material of the workpiece as the proposed algorithm is not focused to optimize the feed rate, cutting speed, radial and axial depth of cut, but the cutting direction, lead_z and tilt_z angle and scallop value as the geometrically defined parameter related to the real shape of the surface. ENAW7075 aluminum alloy was chosen as the material to machine the testing surface because it is necessary to eliminate the external influences, such as chatter, high or uneven distrubution of cutting forces, to machine the surface as the best imprint of the toolpath in the NC program. The difficult to cut material could affect this test so using the material ENAW7075 reduces the possibility of bad influence of cutting forces on the final accuracy of the machined surface. The testing surface was machined using the previously mentioned five axis machine tool—Fig. 13. The toolpath setting (obtained in subchapter 3.4)

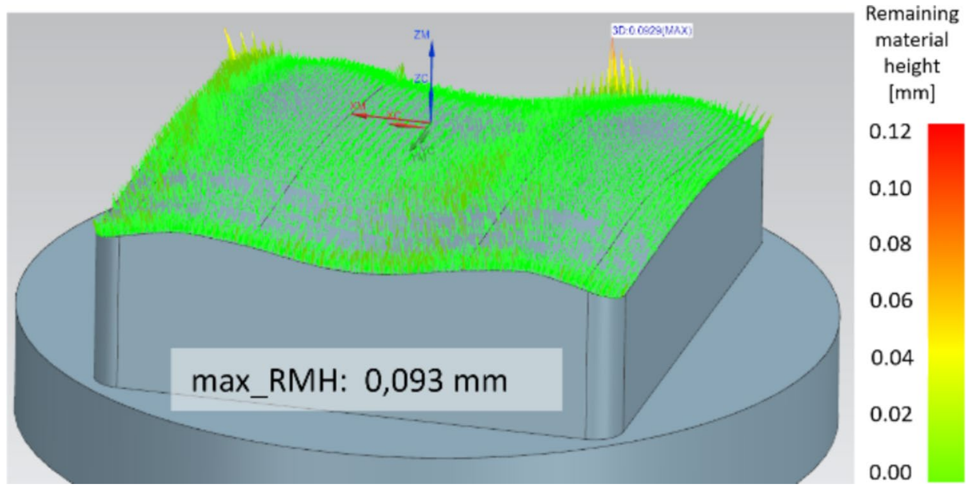

Fig. 14 Max. remaining material height after scallop optimization

Table 2 Comparison of milling times obtained from CAM and from machine tool controller (CNC)

Index [–]	[°]	[°]	[°]	Time CAM [min]	Time CNC [min]	Time ratio [–]
1	0	10	5	26.49	27.23	1.03
2	0	10	10	26.83	27.72	1.03
3	0	20	5	25.26	26.02	1.03
4	0	20	10	25.97	26.73	1.03
5	0	30	5	24.36	25.12	1.03
6	0	30	10	24.69	25.47	1.03
7	45	10	5	26.90	27.83	1.03
8	45	10	10	26.99	28.00	1.04
9	45	20	5	26.35	27.25	1.03
10	45	20	10	26.17	27.07	1.03
11	45	30	5	25.02	25.90	1.04
12	45	30	10	25.11	25.98	1.03
13	90	10	5	25.55	26.20	1.03
14	90	10	10	25.66	26.38	1.03
15	90	20	5	25.20	25.85	1.03
16	90	20	10	25.48	26.13	1.03
17	90	30	5	24.15	24.80	1.03
18	90	30	10	24.39	25.03	1.03

Table 3 Minimum and maximum milling time

	Index [–]	[°]	[°]	[°]	Time CAM [min]
Min time setup:	17	90	30	5	24.15
Max time setup:	8	45	10	10	26.99
Time difference [min]	–	–	–	–	2.84
Time difference [%]	–	–	–	–	11

was used to mill the surface, as seen in Fig. 16 along with the toroidal tool. The same cutting conditions that are presented in chapter 3.2 are used for real machining.

The machined surface was then scanned using an Alicona IFM G5 optical microscope (Alicona Bruker GmbH). IFM stands for infinite focus measurement function. Figure 17 shows the testing surface clamped on the optical microscope. The main scan of the entire surface was made with 50 times magnification. Surface scanning is performed to verify that the maximum remaining material height predicted by simulation in the CAM system has been achieved.

The comparison of the remaining material heights obtained by prediction from the CAM system and by 3D scan from the optical microscope can be seen in Fig. 18. It is clear that the two sections with the greatest remaining material heights are at the same places in both cases. Figure 19 shows detailed views of both sections. These detailed scans are obtained with 100 times magnification. It is clear that the maximum values are the same as predicted in the CAM system. Thus, the algorithm has been verified.

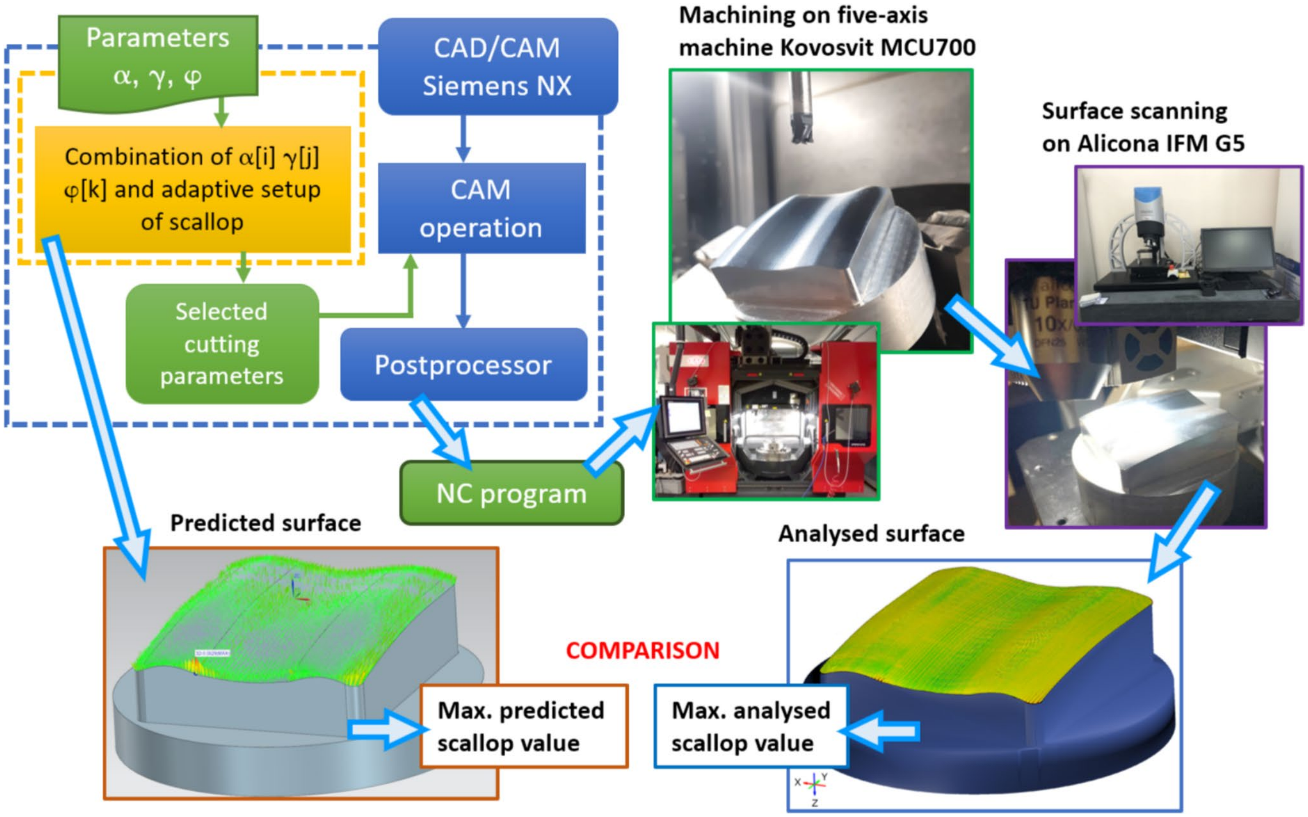

Fig. 15 Verification process to compare the max. predicted scallop value with the max. analyzed scallop value

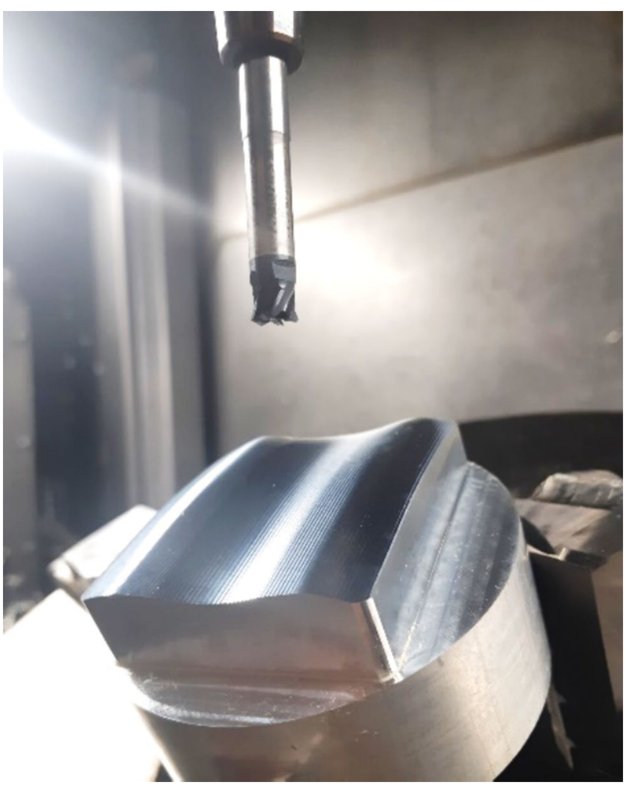

Fig. 16 Tool with the machined surface

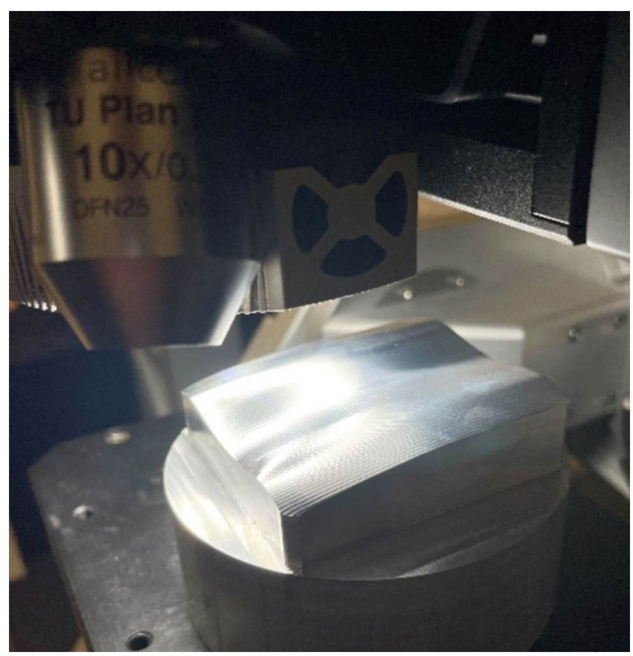

Fig. 17 Surface scanning

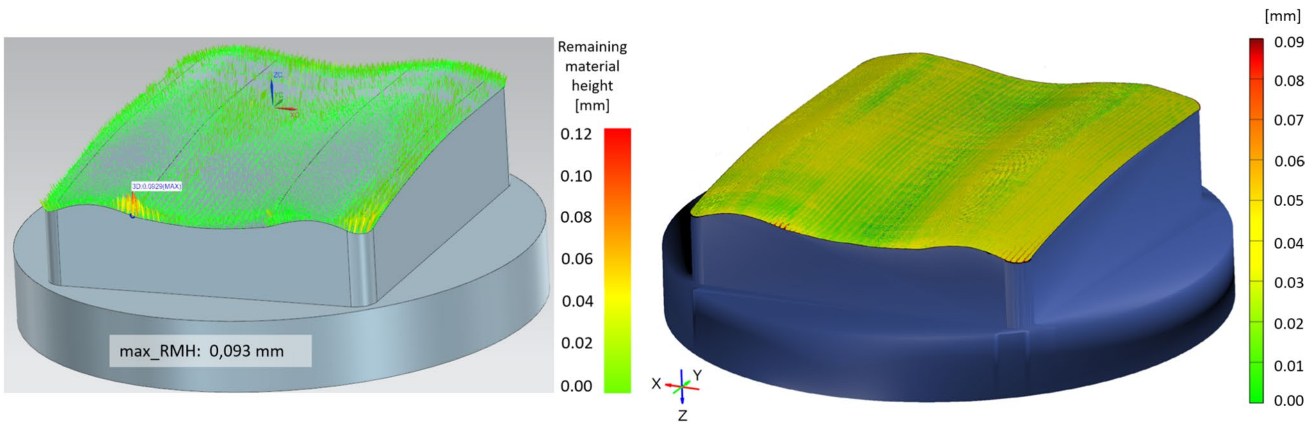

Fig. 18 Comparison of remaining material height (left – analysis from CAM system, right – analysis from 3D scan)

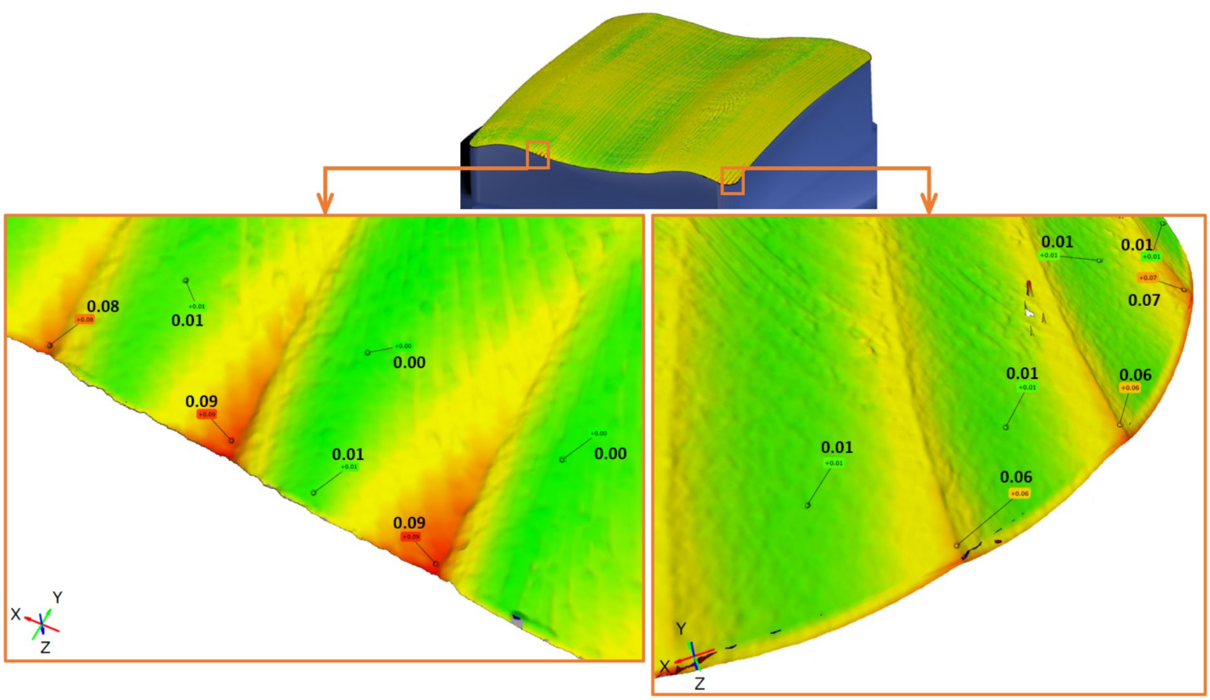

Fig. 19 Detailed scans of two surface sections (left – the section with the greatest remaining material height, right – the section with the second greatest remaining material height) with height of remaining material in [mm]

3.6 Influence on toolpath calculation time

Another important aspect for the real use of the proposed function (user-friendliness) is detecting the influence on the toolpath calculation time. Therefore, the toolpath generation time was measured when the proposed function was used, while two input parameter range settings were tested, with the first setting leading to a total of 18 combinations and the second setting 30 combinations (see Table 4).

The toolpath generation time results in Table 4 show that the total calculation time using the algorithm is 6.1 min for 18 input parameter combinations and 10.1 min for 30 input parameter combinations, which is a total of 0.34 min of generation per toolpath. This is a very favorable finding, since at this time the technologist would not have had time to test such a large number of toolpaths, and moreover it is a minimal time requirement.

Furthermore, the influence of the toolpath calculation tolerance (the lower the tolerance, the more toolpath points) on the calculation time of the proposed algorithm was also tested. For this test, parameter settings leading to 30 combinations were used and a total of 5 different toolpath tolerance settings were selected (see Table 5). As expected, it

Table 4 Influence of toolpath count on calculation time

Toolpath count [−]	[°] (min–max)/incr	[°] (min–max)/incr	[°] (min–max/incr	Total calcul. time [min]	Calcul. time per toolpath [min]
18	(0–180)/60	(10–30)/10	(5–10)/5	6.1	0.34
30	(0–180)/36	(10–30)/10	(5–10)/5	10.1	0.34

Table 5 Influence of toolpath tolerance on calculation time

Setup: (30 toolpaths)	[°] (min–max)/incr.: (0–180)/36	
	[°] (min–max)/incr.: (10–30)/10	
	[°] (min–max)/incr.: (5–10)/5	
toolpath tol. [mm]	Total calcul. time [min]	Calcul. time per toolpath [min]
0.001	10.1	0.34
0.005	7.5	0.25
0.01	6.6	0.22
0.05	5.5	0.18
0.1	5.4	0.18

was found that the lower the tolerance, the longer the total algorithm calculation time and calculation time per toolpath. However, the time difference is not dramatic enough to warrant degrading toolpath quality by reducing the tolerance. However, for large areas with long toolpaths, the proposed function can first be carried out with a worse toolpath tolerance which determines the best setting of the input parameters, and then a narrower range of input parameters with a lower tolerance can be chosen. This will use the time effectively.

Based on these results, the applicability of the newly designed method for searching for the effective setting of cutting parameters (cutting angle, tilt_z angle, lead_z angle, and scallop) to achieve the lowest machining time in point milling operation was verified. As this presented method is based on combinatorial attitute which is a common idea, it has been opted for this to facilitate its immediate application in practice, particularly for integration into a CAM system and for verifying its practical benefits. Future research could explore the implementation of genetic or generative approaches to optimize the process of the cutting conditions selection, particularly in the context of discrete value sets, as in the present case. Several methodologies, such as variational autoencoders (VAE) for discrete data, generative adversarial networks (GANs) adapted for discrete spaces, genetic algorithms, and reinforcement learning, could be considered as so as in [42–44] or [45]. However, it is essential to first assess the effectiveness of these methods to determine their applicability and potential to be effectively used in this specific task.

4 Conclusion

In this paper, a method was proposed, which makes it possible to find the best setting of specific cutting toolpath parameters when using a milling tool with circular cutting edge to achieve the lowest possible milling time while obtaining the required scallop value of 3 + 2 axis milling, regardless of the toolpath calculation core. The proposed algorithm can effectively use the range of input parameters that has been set and determine the cutting direction angle, tool orientation (two angles defining tool axis vector) and scallop calculation value (in fact by adjusting the stepover, but controlled by the scallop_OPT value) in a combination resulting in the lowest machining time while keeping the toolpath native to the specific CAM system. The method can be used not only for toolpaths based on the parallel planes pattern, but also for other patterns (such as spiral, offset) when using only tool orientation and the max. scallop value as input parameters. Using a machined shape surface as an example, it was verified that within the setting of the input parameters, the algorithm first found a setting that yields a time saving of 11% compared to the most time-consuming parameter setting variant, and in the second (adaptive) phase, the algorithm further optimized the scallop calculation value to obtain another time saving of 19%, while the required maximum height of the remaining material on the workpiece surface was analyzed. At the same time, this algorithm was implemented into a common CAM system as a library, designed in a user-friendly way. Furthermore, it was verified that the computational time of the algorithm is 0.34 min for one toolpath (one combination of input parameters) for a toolpath tolerance of 0.001 mm, which is a very favorable time. The algorithm can therefore be used on various surfaces with complex shapes (even when applied to surfaces machined in parts), which provides very significant support to the technologist in the facilitation of routine tasks associated with the selection of geometrically defined parameters and thus allows him to focus on truly sophisticated tasks, such as the selection of parameters, which also require experience (such as feed per tooth, cutting speed, tolerance).

Author contribution Petr Vavruska: conceptualization, methodology, project administration, writing—original draft; Filip Kasik: software, visualization, data curation; Jan Lomicka: methodology, data curation; Adam Cermak: investigation, data curation, formal analysis.

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Data availability The data underlying this paper will be shared on reasonable request to the corresponding author.

Declarations

Competing interests The authors declare no competing interests.

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